

**Objectives:**

- Define relationships between  $f(x)$ ,  $f'(x)$  and  $f''(x)$ .
- Use information from  $f(x)$  to graph  $f'(x)$ .

**What does  $f(x)$  tell us about  $f'(x)$ ?**

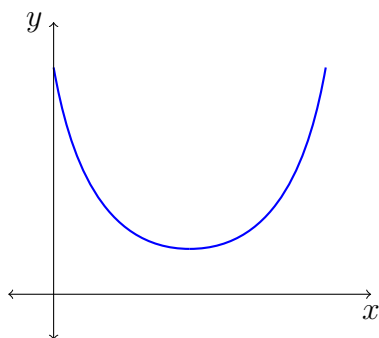
If  $f(x)$  is increasing at  $x = a$ , then  $f'(a)$  is positive.

If  $f(x)$  is decreasing at  $x = a$ , then  $f'(a)$  is negative.

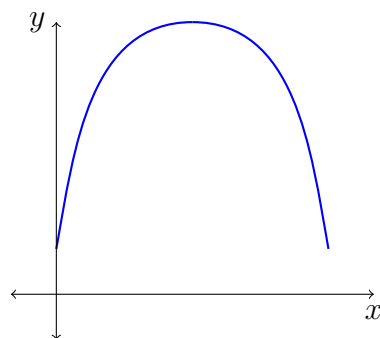
If  $f(x)$  has a horizontal tangent line at  $x = a$ , then  $f'(a) = \underline{0}$ .

*Note:* If  $f(x)$  is discontinuous at  $a$ , has a corner/cusp at  $a$ , or has a vertical tangent line at  $a$ , then  $f'(a)$  is undefined.

**What does  $f(x)$  tell us about  $f''(x)$ ?**



Concave up



Concave down

If  $f(x)$  is concave up,  $f'(x)$  is increasing, so  $f''(x)$  is positive.

If  $f(x)$  is concave down,  $f'(x)$  is decreasing, so  $f''(x)$  is negative.

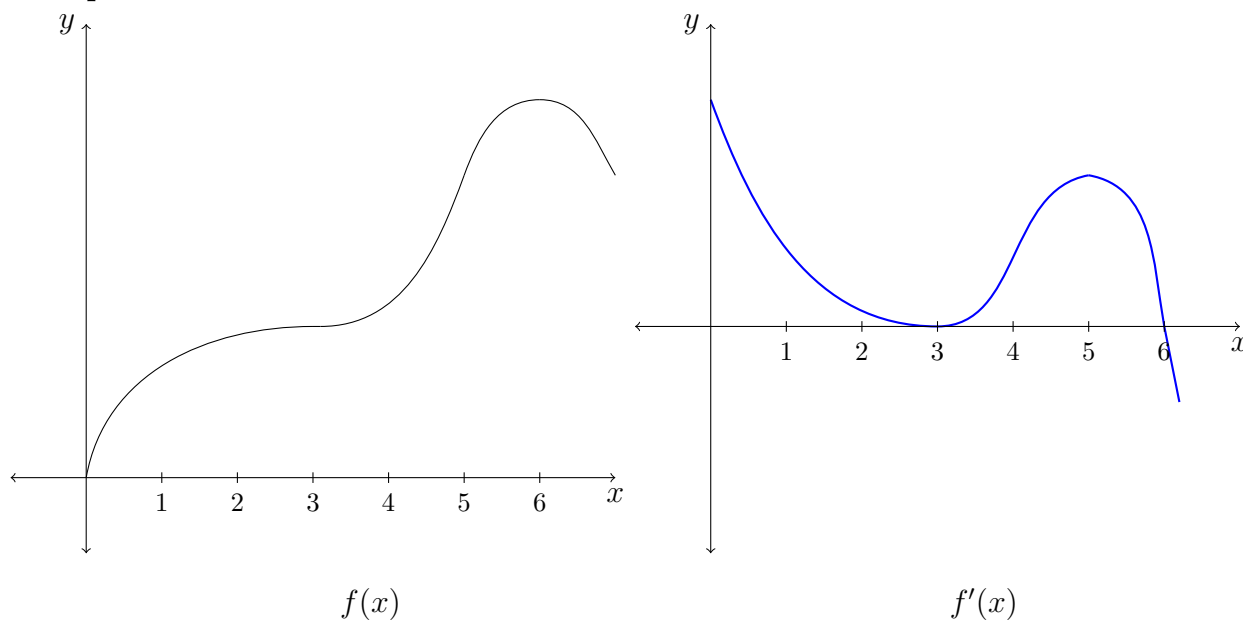
**Summary**

First, look for points where the derivative or second derivative is zero. Then consider where  $f'(x)$  and  $f''(x)$  are positive or negative, according to the following patterns:

$f(x)$	<u>increasing</u>	<u>decreasing</u>
$f'(x)$	<u>+</u>	<u>-</u>

$f(x)$	<u>concave up</u>	<u>concave down</u>
$f'(x)$	<u>increasing</u>	<u>decreasing</u>
$f''(x)$	<u>+</u>	<u>-</u>

Example:



	$x \in (1, 3)$	$x = 3$	$x \in (3, 6)$	$x = 6$	$x \in (6, 7)$
$f(x)$	increasing	horizontal tangent	increasing	horizontal tangent	decreasing
$f'(x)$	+	0	+	0	-

	$x \in (1, 3)$	$x \in (3, 5)$	$x \in (5, 7)$
$f(x)$	concave down	concave up	concave down
$f'(x)$	decreasing	increasing	decreasing
$f''(x)$	-	+	-

We'll use these basic rules in today's class activity. The solutions to the activity will be posted on the course website - I would recommend adding at least some of those examples to your notes.